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QUANTUM FIELD THEORY WITHOUT DIVERGENCE A

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ABSTRACT. We anew explain the meaning of negative energies in the relativistic theory. On the basis we present two new conjectures. According to the conjectures, particles have two sorts of existing forms which are symmetric. From this we present a new Lagrangian density and a new quantization method for QED. That the energy of the vacuum state is equal to zero is naturally obtained. From this we can easily determine the cosmological constant according to experiments, and it is possible to correct nonperturbational methods which depend on the energy of the ground state in quantum field theory.

1. INTRODUCTION

There are the following five problems to satisfactorily solve in the given quantum field theory (QFT).

1. The issue of the cosmological constant.
2. The problem of divergence of Feynman integrals with loop diagrams.
3. The problem of the origin of asymmetry in the electroweak unified theory.
4. The problem of triviality of φ^4-theory.
5. The problems of dark matter and the origin of existence of huge cavities in cosmos.

In brief, we present a consistent QFT without divergence, give a fully method evaluating Feynman integrals (see the second and third papers), give possible solutions to the five problems in a unified basis to reexplain the physics meanings of negative energies.

There is an inconsistency in the convintional QFT.

As is well known, before redefining a Hamilton H as a normal-ordered product, the energy E_0 of the vacuum state is divergent. Because we may arbitrarily choose the zero point of energy in QFT, we can redefine E_0 to be zero. This is, in fact, equivalent to demand

$$\{a_{\mathbf{p}s}, a^+_{\mathbf{p}s}\} = \{b_{\mathbf{p}s}, b^+_{\mathbf{p}s}\} = [c_{\mathbf{k}\lambda}, c^+_{\mathbf{k}\lambda}] = 0,$$

in the convintional QFT. But in fact these commutation relations are equal to 1, and in other cases, e.g. in propagators, they must also be 1. Thus the given QFT is not consistent.

Divergence of Feynman integrals with loop diagrams seems to have been solved by introducing the bare mass and the bare charge or the concepts equivalent to

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them. But both bare mass and charge are divergent and unmeasured, thus QFT is still not perfect. In order to overcome the shortcomings, people have tried many methods. For example, G. Scharf attempted to solve the difficulty by the causal approach [1]. Feynman integrals with loop diagrams are not divergent in some supersymmetric theories. But the supersymmetry theory lacks experiment foundations. In fact, there should be no divergence and all physical quantities should be measurable in a consistent theory.

According to the given generalized electroweak unified models which are left-right symmetric before symmetry spontaneously breaking, asymmetry is caused by symmetry spontaneously breaking. In such models there must be many unknown particles with massive masses. Such models are troublesome and causes many new problems. Hence the origin of asymmetry in the electroweak unified theory should still be explored.

In order to introduce the present theory, we first discuss measurement of an energy. For a measurable energy, in fact, there is the following conjecture.

Conjecture 1. Any measurable energy E of a physical system must be a difference between two energies E_i' and E_j' belonging respectively to two states $\mid E_i' \rangle$ and $\mid E_j' \rangle$, i.e.,

$$E = \langle E_i' \mid H \mid E_i' \rangle - \langle E_j' \mid H \mid E_j' \rangle = E_i' - E_j', \tag{1.1}$$

where $\mid E_j' \rangle$ is such a state into which $\mid E_i' \rangle$ can transit by radiating the energy E.

It is seen from the conjecture 1 that in order to determine a measurable energy E we must firstly determine two states $\mid E_j' \rangle$ and $\mid E_i' \rangle$, and only when $\mid E_i' \rangle$ can transit into $\mid E_j' \rangle$ by radiating the energy E, E is measurable. Let $E'_{\min}$ be the minimal energy of a system,an energy E of the system can be defined as $E = E_j' - E'_{\min}$. It is obvious that $E \geqslant 0$. $E'_{\min} =?$ $E'_{\min} = 0$ is necessary. According to the general relativity

$$m_g = m_i = E, \tag{1.2}$$

where m_g and m_i are the gravitational mass and the inertial mass corresponding to E, respectively. If $E'_{\min} \neq 0$, $E'_{\min}$ will cause a gravitational effect. Thus $E'_{\min}$ is measurable (let the cosmological constant be given). If $E'_{\min}$ is a measurable energy, $E'_{\min} \geqslant 0$. If $E'_{\min} > 0$, there must be another state $\mid E''_{\min} \rangle$ with its energy $E''_{\min} < E'_{\min}$, thus $\mid E'_{\min} \rangle$ is no longer the ground state. Hence we should have

$$E'_{\min} = 0. \tag{1.3}$$

But before Hamiltonian operator is defined as a normal-ordered product, the energy $E_0 = E'_{\min}$ is divergent, thus the convintional QFT is inconsistent with the general theory of relativity and cause the issue of cosmological constant. Of course, if the general theory of relativity is not considered, $E'_{\min}$ is unmeasured and undertermined.

It is seen from the above mentioned that the conventional QFT is not perfect and should be corrected.

We introduce the present theory as follows.

1. A new Lagragian density.

The physics basis of the present theory is to reexplain the physics meanings of negative energies. The relativistic theory is very perfect, and existence of negative energies is its essential character. Any existence must depend on its existing conditions. As an ancient Chinese philosopher said, nothing in nothing is just some existence; existence in existence is just some nothing. We think that positive energies and negative energies are depend on each other, not only are not negative energies a difficulty, but have profound physical meanings. Existence of antiparticles is only, in fact, a result of particle-inversion symmetry, and do not reveal the essence of negative energies. For example, a pure neutral particle or a pure neutral world has also negative-energy states. We guess that negative-energy states correspond another sort of existence forms of matter, and from this present the following conjecture.

: Conjecture 2. The Lagrangian density describing the world is

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{W.} \tag{1.4}$$

$\mathcal{L}_F$ and $\mathcal{L}_W$ are independent from each other in classical meanings and symmetric, where $\mathcal{L}_F$ is the same as the Lagrangian density in the conventional QFT in the classical meanings and describes given particles, and $\mathcal{L}_W$ describes unknown particles corresponding to the particles in $\mathcal{L}_F$.

According to the conjecture, every particle in $\mathcal{L}_F$ is accordant with a particle in $\mathcal{L}_W$, and the properties of the two particles are the same, e.g., there are two sorts of electrons, F-electron and W-electron. That $\mathcal{L}_F$ and $\mathcal{L}_W$ are independent from each other implies that there is no coupling between a field in $\mathcal{L}_F$ and a field in $\mathcal{L}_W$, but after quantization, $\mathcal{L}_F$ and $\mathcal{L}_W$ are dependent on each other. Thus, the two sorts of energies corresponding to $\mathcal{L}_F$ and $\mathcal{L}_W$ are respectively conservational, a real particle in $\mathcal{L}_F$ cannot transform into a real particle in $\mathcal{L}_F$, and vice versa. But the two sorts of virtual particles can transform from each into other.

We explain the necessity to present the second conjecture in relativistic quantum mechanics (RQM) framework as follows.

If an electron e^- and a positron e^+ are complete symmetric, there should be $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{W.}$

As is well known, the free electron equation

$$(\gamma_\mu \partial_\mu + m)\psi = 0$$

has negative-energy and positive-energy solutions. We denote an unoccupied negative-energy state by $\mid E_j^-, e^-, 0\rangle$, an occupied negative-energy state by $\mid E_j^-, e^-, 1\rangle$ and a occupied positive-energy by $\mid E_j^-, e^-, 1\rangle$. According to the conventional RQM, existence of $\mid E_j^-, e^-, 0\rangle$ is unconditioned. Thus the negative-energy difficulty appears. In order to overcome the difficulty, Dirac presented his hole theory, i.e., all negative-energy states are occupied and $\mid E_j^-, e^-, 0\rangle$ is equivalent to a positron state $\mid E_j^+, e^+, 1\rangle$. According to Dirac's theory, e^- and e^+ are not symmetric. The conventional QED derived from the theory has the two difficulties, i.e., E_0 and Feynman integrals with loop are divergent.

In order to overcome the difficulties, we suppose that there is the following process

$$| E_j^-, e^-, 0\rangle \rightleftarrows | E_j^-, e^-, 1\rangle \mid E_j^+, e^+, 1\rangle. \tag{1.5}$$

Because the original Lagrangian density $\mathcal{L}_F$ only descrbes e^-−states $| E_j^\pm, e^-, 1\rangle$ and cannot describe e^+−states $| E_j^\pm, e^+, 1\rangle$, $\mathcal{L}_F$ should be corrected to $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W$, where $\mathcal{L}_W$ describes e^+−states $| E_j^\pm, e^+, 1\rangle$. Analogously to Dirac's theory, now it is necessary in the RQM framework to suppose all $| E_j^-, e^-, 0\rangle$ and $| E_j^-, e^+, 0\rangle$ to be occupied. Thus, e^- and e^+ are symmetric and have respectively two sorts of existence forms, i.e., e^+ can exist in $| E_j^-, e^-, 0\rangle$ or $| E_j^+, e^+, 1\rangle$ and e^- can exist in $| E_j^-, e^+, 0\rangle$ or $| E_j^+, e^-, 1\rangle$, here we regard $| E_j^-, e^\pm, 1\rangle$ as a part of the ground state. The two sorts of existence forms can transform from one into other, e.e., (1.5) and

$$| E_j^-, e^+, 0\rangle \rightleftarrows | E_j^-, e^+, 1\rangle \mid E_j^+, e^-, 1\rangle. \tag{1.6}$$

We will see that $E_0 = 0$ and all Feynman integrals are convergent in the new QED derived from $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W$.

In fact, we may also suppose that existence of $| E_j^-, e^-, 0\rangle$ and $| E_j^-, e^+, 0\rangle$ is not unconditioned, but just conditioned as existence of $| E_j^+, e^-, 0\rangle$ and $| E_j^+, e^+, 0\rangle$. Thus when $| E_j^-, e^-, 0\rangle$ or $| E_j^-, e^+, 0\rangle$ does not exist, e^- or e^+ cannot transit into the negative-energy states $| E_j^-, e^-, 0\rangle$ or $| E_j^-, e^+, 0\rangle$, and it is unnecessary to suppose all negative-energy states $| E_j^-, e^-, 0\rangle$ and $| E_j^-, e^+, 0\rangle$ to be occupied.

When ψ and A_μ in $\mathcal{L}_F$ and $\underline{\psi}$ and $\underline{A}_\mu$ in $\mathcal{L}_W$ are regarded as the classical fields or the coupling coefficients g_f and the electromagnetic masses m_{ef} in $\mathcal{L}_F$, g_w and m_{ew} in $\mathcal{L}_W$ are regarded as constants, $\mathcal{L}_F$ and $\mathcal{L}_W$ will be independent of each other. In this case, except $E_0 = 0$, all results obtained by the present theory will be same as those obtained by the conventional theory.

2. Transformation operators and a new method quantizing fields.

Because particles can exist in the two sorts of forms, we can define transformation operators which transform a F-state into a W-state or a W-state into a F-state, and can quantize fields by the transformation operators replacing creation and annihilation operators in the conventional QFT. Thus it is necessary that g_f and m_{ef} respectively become operators G_F and M_F to be determined by $\mathcal{S}_w$, and g_w and m_{ew} respectively become operators G_W and M_W to be determined by $\mathcal{S}_f$, here $\mathcal{S}_w$ and $\mathcal{S}_f$ are the scattering operators respectively determined by $\mathcal{L}_W$ and $\mathcal{L}_F$. G_F and M_F multiplied by field operators ψ and A_μ become the coupling coefficients $g_f(p_2, p_1)$ and mass $m_{ef}\ (p)$ determined by scattering amplitude $\langle W_f \mid S_w \mid W_i \rangle$, and G_W and M_W multiplied by field operators $\underline{\psi}$ and $\underline{A}_\mu$ become the coupling coefficients $g_w(p_2, p_1)$ and mass $m_{ew}\ (p)$ determined by scattering amplitude $\langle F_f \mid S_f \mid F_i \rangle$. Thus after quantization, $\mathcal{L}_F$ and $\mathcal{L}_W$ will be dependent on each other.

3. Two sorts of corretions.

In the conventional QED, there are two sort of parameters, e.g., the physical charge and the bare charge, and one sort of corrections originating $\mathcal{S}$ equivalent to $\mathcal{S}_f$. In contrast with the given QED, there are only one sort of parameters defined at so-called subtraction point q_2, q_1 and $q\prime$, i.e., $g_f(q_2, q_1) = g_w(q_2, q_1) = g_0$ and $m_{ef}(q) = m_{ew}(q) = m_{e0}$, and two sorts of corretions originating from $\mathcal{S}_w$ and $\mathcal{S}_f$ to scattering amplitudes, g_0 and m_{e0}.Thus $\mathcal{L}_F$ and $\mathcal{L}_W$ together determine the loop-diagram corrections. When n-loop corrections originating from $\mathcal{S}_f$ and $\mathcal{S}_w$ are simultaneously considered, the integrands causing divergence in $\langle F_f \mid S_f \mid F_i \rangle$ or $\langle W_f \mid S_w \mid W_i \rangle$ will cancil each other out, hence all Feynman integrals are convergent, e.g.,

$$g_f^{(1)}(p_2, p_1) = g_{ff}^{(1)}(p_2, p_1) + g_{fw}^{(1)}(q_2, q_1)$$

is finite and $g_f^{(1)}(q_2, q_1) = 0$, where $g_{ff}^{(1)}$ originates from $\mathcal{S}_f$ and $g_{fw}^{(1)}$ originates from $\mathcal{S}_w$, and the superscript (1) denotes 1-loop correction. Thus it is unnecessary to introduce counterterms and regularization. We give a complete Feynman rules to evaluate Feynman integrals by the new concepts (see second and third papers).

It should be pointed that in the meaning of perturbation theory, because the coupling coefficients and masses will be corrected by n-loop diagrams, we cannot give a absolutely precise $\mathcal{L}_F$ and $\mathcal{L}_W$, and can only give the precise $\mathcal{L}_F^{(0-loop)}$ and $\mathcal{L}_W^{(0-loop)}$ at the subtraction point or approximate to tree diagrams. Of course, by such $\mathcal{L}_F^{(0-loop)}$ and $\mathcal{L}_W^{(0-loop)}$ we can obtain scattering amplitudes approximate arbitrary n-loop diagrams.

4. $\langle 0 \mid H \mid 0 \rangle \equiv E_0 = E_{0F} + E_{0F} = 0$ is naturally derived, thereby we can easily determine the cosmological constant according to data of astronomical observation, and it is possible to correct nonperturbational methods which depend on the energy of the ground state in QFT.

5. Generalizing the present theory to the electroweak unified theory, we will see a possible origin of symmetry breaking. Accoding to this model, the world is symmetric on principle (i.e., $\mathcal{L} = \mathcal{L}_W + \mathcal{L}_F$ is symmetric), but the world observed by us is asymmetric since $\mathcal{L}_W$ or $\mathcal{L}_F$ is asymmetric. In this model there is no unknown particle with a massive mass (see the third paper).

6. Because there is no interaction between the two sorts of matter by a given quantizable field. Only possibility is that there is repulsion or gravitation of the two sorts of matter. Because the sort of matter described by $\mathcal{L}_W$ is one new sort of matter, it is impossible from theory to determine that there is what sort of interaction. We can only determine the new sort of interaction from data of astronomical observation. If the new interaction is repulsion, it is possible that the new interaction is the reason for existence of the phenomena of huge cavity. If the new interaction is gravitation, it is possible that the new sort of matter is the candidate for dark matter. It is also possible that there is new and more important relationship between matter with negative energy and matter with positive energy.

7. The new QFT will also give a possible solution for the problem of triviality of φ^4-theory.

8. Two sorts of transformation.

Accoding to the present theory, there are two sorts of transformation.

The first sort of transformation must correspond to a coupling term of field operators, e.g., $ig\overline{\psi}\gamma_\mu A_\mu\psi$ determines the transformation $e^+ + e^- \to \gamma + \gamma$. The sort of transformation is measurable.

The second sort of transformation is defined by the transformation operators as $\mid \underline{a}_{\mathbf{p}s}\rangle \leqslant a_{\mathbf{p}s} \mid$ and cannot correspond to any coupling term of quantizable fields . The processes determined by the sort of transformation, e.g.,

$$\mid \underline{a}_{\mathbf{p}s}\rangle \leqslant a_{\mathbf{p}s} \mid\mid a_{\mathbf{p}s}\rangle = \mid \underline{a}_{\mathbf{p}s}\rangle,$$

cannot be measured. The sort of transformation can only be potential and is realized by the virtual-particle processes. Existing reasons of the sort of transformation are that by it we can eliminate divergence of E_0 and Feynman integrals with loop diagrams, explain the left-right asymmetry and some phenomena of the cosmos, and so on.

By the conventional creation and annihilation operators in the given QFT we can also obtain the similar results provided we Suppose $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W$ and that g_F and m_F are determined by $\mathcal{S}_W$ and g_W and m_W are determined by $\mathcal{S}_F$ (of course, in this case this conjecture is not natural). It is also possible to obtain the same results but that F-particles possess positive energies and W-particles possess negative energies, provided $\mathcal{L}_F$ and $\mathcal{L}_W$ are independent of each other in classical meanings.

The present theory contains three parts. The first part takes QED as example to illuminate the method to reconstruct QFT, and give the solutions of the issue of the cosmological constant and the problem of divergent Feynman integrals in QED. The part is composed of the present paper and the following two papers. The second part discusses the problem of triviality of φ^4-theory. The third part discusses the problem of the origin of asymmetry in the electroweak unified theory.

Quantization for free fields will be discussed in the present paper. The outline of this paper is as follows. Section 2: Lagrangian density;Section 3: quantization for free fields; Section 4: *The energies and charges of particles; Section 5: Subsidiary condition; Section 6: The equations of motion;* Section 7:The physical meanings of that the energy of the vacuum state is equal to zero; Section 8 is conclusion. Section 9: Appendix A.

2. Lagrangian density

We suppose the Lagrangian density for the free Dirac fields and the Maxwell fields to be

$$\mathcal{L}_0 = \mathcal{L}_{F0} + \mathcal{L}_{W0}, \tag{2.1}$$

$$\mathcal{L}_{F0} = -\overline{\psi}(x)(\gamma_\mu\partial_\mu + m)\psi(x) - 12\partial_\mu A_\nu\partial_\mu A_\nu, \tag{2.2}$$

$$\mathcal{L}_{W0} = \overline{\underline{\psi}}(x)(\gamma_\mu \partial_\mu + m)\underline{\psi}(x) - 12\partial_\mu \underline{A}_\nu \partial_\mu \underline{A}_\nu, \tag{2.3}$$

(2.2) and (2.3) imply the Lorentz gauge to be already fixed . The difference between (2.1) and the corresponding Lagrangian density in the convertional QED is $\mathcal{L}_{W0}$ which describes motion of particles existing in the other form. We call ψ, A_μ, $\underline{\psi}$ and $\underline{A}_\mu$ the F-electron field, the F-photron field, and the W-electron field, the W-photron field, respectively. The conjugate fields corresponding to them are respectively

$$\pi_\psi = \partial \mathcal{L} \partial \dot{\psi} = i\psi^+, \qquad \pi_\mu = \partial \mathcal{L} \partial \dot{A}_\mu = \dot{A}_\mu, \tag{2.4}$$

$$\pi_{\underline{\psi}} = \partial \mathcal{L} \partial \dot{\underline{\psi}} = -i\underline{\psi}^+, \qquad \underline{\pi}_\mu = \partial \mathcal{L} \partial \dot{\underline{A}}_\mu = \dot{\underline{A}}_\mu. \tag{2.5}$$

From the Noether′s theorem, we obtain the conservational quantities

$$H_0 = H_{F0} + H_{W0}, \tag{2.6}$$

$$H_{F0} = \int d^3x[\psi^+ \gamma_4 \left(\gamma_j \partial_j + m\right) \cdot \psi + 12 \left(\dot{A}_\mu \cdot \dot{A}_\mu + \partial_j A_\nu \cdot \partial_j A_\nu\right)] \ , \tag{2.7}$$

$$H_{W0} = -\int d^3x[(\underline{\psi}^+ \gamma_4 (\gamma_j \partial_j + m) \cdot \underline{\psi} - 12(\dot{\underline{A}}_\mu \cdot \dot{\underline{A}}_\mu + \partial_j \underline{A}_\nu \cdot \partial_j \underline{A}_\nu)], \tag{2.8}$$

$$Q = Q_F + Q_W, \qquad Q_F = \int d^3x \psi_0^+ \cdot \psi, \qquad Q_W = \int d^3x \underline{\psi}_0^+ \cdot \underline{\psi}_0. \tag{2.9}$$

The global gauge transformations corresponding to (2.9) are

$$\psi \to \psi' = e^{i\alpha}\psi, \ \ \underline{\psi} \to \underline{\psi}' = e^{-i\alpha}\underline{\psi} \ . \tag{2.10}$$

The Euler-Lagrange equations of motion being derived from Hamilton's variational principle are

$$i\partial\partial t \psi = \hat{H}_0 \psi, \qquad \Box A_\mu = 0, \tag{2.11}$$

$$i\partial\partial t\underline{\psi} = \hat{H}_0\underline{\psi}, \qquad \square\underline{A}_\mu = 0, \tag{2.12}$$

where $\hat{H}_0 = \gamma_4\left(\gamma_j\partial_j + m\right)$. It is seen from (2.11)-(2.12) that although $\mathcal{L}_{F0} \neq \mathcal{L}_{W0}$, the equations satisfied by $\underline{\psi}$ and $\underline{A}_\mu$ are the same as those satisfied by ψ and A_μ, respectively. This implies that for a relativistic physical system, only equations of motion are insufficient for corrective description of all properties of the system. A complete Lagrangian density is very necessary.

When ψ, etc., are regarded as the classical fields and

$$\partial_\mu A_\mu = \partial_\mu \underline{A}_\mu = 0, \tag{2.13}$$

ψ and $\underline{\psi}$ can be expanded in terms of the complete set of plane-wave solutions

$$1\sqrt{V}u_{\mathbf{p}s}e^{ipx}, \qquad 1\sqrt{V}v_{\mathbf{p}s}e^{-ipx}, \qquad s = 1, 2, \tag{2.14}$$

where $px = \mathbf{px} - E_\mathbf{p}t$, $E_\mathbf{p} = \sqrt{\mathbf{p}^2 + m^2}$, and A_μ and $\underline{A}_\mu$ can be expanded in terms of the complete set

$$1\sqrt{2\omega_\mathbf{k}V}e^\lambda_{\mathbf{k}\mu}e^{\pm ikx}, \tag{2.15}$$

where $kx = \mathbf{kx} - \omega_\mathbf{k}t, \omega_\mathbf{k} = \mid \mathbf{k} \mid$, $\lambda = 1, 2,$. To get a completeness relation, it is necessary to form a guartet of orthonormal 4-vectors[2].

$$\begin{aligned} e^1_k &= \left(\varepsilon^1_\mathbf{k}, 0\right), e^2_k = \left(\varepsilon^2_\mathbf{k}, 0\right), e^3_k = -\left[k + \eta\left(k\eta\right)\right] \diagup k\eta, \\ \eta &= \left(0, 0, 0, i\right), e^4_k = i\eta, \varepsilon^{1,2}_\mathbf{k} \cdot \mathbf{k} = \mathbf{0}. \end{aligned}$$

Moreover, all four vectors are normalized to 1, i.e.,

$$e^\lambda_\mathbf{k}e^{\lambda'}_\mathbf{k} = \delta_{\lambda\lambda'}, \qquad \sum_{\lambda=1}^4 e^\lambda_{\mathbf{k}\mu}e^\lambda_{\mathbf{k}\nu} = \delta_{\mu\nu.}$$

3. Quantization for free fields

We now regard ψ etc., as quantum fields. ψ, A_μ, $\underline{\psi}$ and $\underline{A}_\mu$ as the solutions of the equations of the quantum fields (2.11) – (2.12) can also be expanded in terms of the complete sets (2.13) and (2.14), respectively, only the expanding coefficients are all operators. Thus we have

$$\psi_0\,(x) = 1\sqrt{V}\sum_{\mathbf{p}s}\left(\underline{I}_{\mathbf{p}} \leqslant a_{\mathbf{p}s}(t) \mid u_{\mathbf{p}s}e^{i\mathbf{px}} + \mid b_{\mathbf{p}s}(t) \geqslant \underline{I}_{(-\mathbf{p})}v_{\mathbf{p}s}e^{-i\mathbf{px}}\right), \tag{3.1}$$

$$A_{0\mu}\,(x) = 1\sqrt{V}\sum_{\mathbf{k}}1\sqrt{2\omega_{\mathbf{k}}}\sum_{\lambda=1}^{4}e^{\lambda}_{\mathbf{k}\mu}\left(\underline{j}_{\mathbf{k}} \leqslant c_{\mathbf{k}\lambda}(t) \mid e^{i\mathbf{kx}} + \mid \overline{c}_{\mathbf{k}\lambda}(t) \geqslant \underline{j}_{(-\mathbf{k})}e^{-i\mathbf{kx}}\right), \tag{3.2}$$

$$\underline{\psi}_0\,(x) = 1\sqrt{V}\sum_{\mathbf{p}s}\left(\mid \underline{b}_{\mathbf{p}s}(t) \geqslant I_{\mathbf{p}}u_{\mathbf{p}s}e^{i\mathbf{px}} + I_{(-\mathbf{p})} \leqslant \underline{a}_{\mathbf{p}s}(t) \mid v_{\mathbf{p}s}e^{-i\mathbf{px}}\right), \tag{3.3}$$

$$\underline{A}_{0\mu}\,(x) = 1\sqrt{V}\sum_{\mathbf{k}}1\sqrt{2\omega_{\mathbf{k}}}\sum_{\lambda=1}^{4}e^{\lambda}_{\mathbf{k}\mu}\left(\mid \overline{\underline{c}}_{\mathbf{k}\lambda}(t) \geqslant j_{(-\mathbf{k})}e^{i\mathbf{kx}} + j_{\mathbf{k}} \leqslant \underline{c}_{\mathbf{k}\lambda}(t) \mid e^{-i\mathbf{kx}}\right), \tag{3.4}$$

$$\pi_{0\psi} = i\psi^{+}_0\,(x) \equiv i\sqrt{V}\sum_{\mathbf{p}s}\left(\mid a_{\mathbf{p}s}(t) \geqslant \underline{I}^{+}_{\mathbf{p}}u^{+}_{\mathbf{p}s}e^{-i\mathbf{px}} + \underline{I}^{+}_{(-\mathbf{p})} \leqslant b_{\mathbf{p}s}(t) \mid v^{+}_{\mathbf{p}s}e^{i\mathbf{px}}\right), \tag{3.5}$$

$$\pi_{0\mu} = \dot{A}_{0\mu}\,(x) \equiv -i\sqrt{V}\sum_{\mathbf{k}}\sqrt{\omega_{\mathbf{k}}2}\sum_{\lambda=4}^{4}e^{\lambda}_{\mathbf{k}\mu}\left(\underline{j}_{\mathbf{k}} \leqslant c_{\mathbf{k}\lambda}(t) \mid e^{i\mathbf{kx}} - \mid \overline{c}_{\mathbf{k}\lambda}(t) \geqslant \underline{j}_{(-\mathbf{k})}e^{-i\mathbf{kx}}\right), \tag{3.6}$$

$$\underline{\pi}_{0\psi} = -i\underline{\psi}^{+}_0(x) \equiv -i\sqrt{V}\sum_{\mathbf{p}s}\left(I^{+}_{\mathbf{p}} \leqslant \underline{b}_{\mathbf{p}s}(t) \mid u^{+}_{\mathbf{p}s}e^{-i\mathbf{px}} + \mid \underline{a}_{\mathbf{p}s}(t) \geqslant I^{+}_{-\mathbf{p}}v^{+}_{\mathbf{p}s}e^{i\mathbf{px}}\right), \tag{3.7}$$

$$\underline{\pi}_{0\mu} = \underline{\dot{A}}_{0\mu}\,(x) \equiv -i\sqrt{V}\sum_{\mathbf{k}}\sqrt{\omega_{\mathbf{k}}2}\sum_{\lambda=1}^{4}e^{\lambda}_{\mathbf{k}\mu}\left(\mid \overline{\underline{c}}_{\mathbf{k}\lambda} \geqslant j_{(-\mathbf{k})}e^{i\mathbf{kx}} - j_{\mathbf{k}} \leqslant \underline{c}_{\mathbf{k}\lambda} \mid e^{-i\mathbf{kx}}\right), \tag{3.8}$$

$$, \mid \overline{c}_{\mathbf{k}\lambda} \geqslant = \begin{cases} \mid c_{\mathbf{k}\lambda} \geqslant, \lambda = 1,2,3, \\ -\mid c_{\mathbf{k}\lambda} \geqslant, \lambda = 4 \end{cases} \tag{3.9}$$

$$\mid \overline{\underline{c}}_{\mathbf{k}\lambda} \geqslant = \begin{cases} \mid \underline{c}_{\mathbf{k}\lambda} \geqslant, \lambda = 1,2,3, \\ -\mid \underline{c}_{\mathbf{k}\lambda} \geqslant, \lambda = 4 \end{cases} , \mid \overline{c}_{\mathbf{k}\lambda} \geqslant = \begin{cases} \mid c_{\mathbf{k}\lambda} \geqslant, \lambda = 1,2,3, \\ -\mid c_{\mathbf{k}\lambda} \geqslant, \lambda = 4 \end{cases}$$

$$
\begin{aligned}
&\leqslant \quad a_{\mathbf{p}s}(t) \mid = \leqslant a_{\mathbf{p}s} \mid e^{-i\omega_{\mathbf{p}}t}, && \mid b_{\mathbf{p}s}(t) \geqslant = \mid b_{\mathbf{p}s} \geqslant e^{i\omega_{\mathbf{p}}t},\\
&\mid \quad a_{\mathbf{p}s}(t) \geqslant = \mid a_{\mathbf{p}s} \geqslant e^{i\omega_{\mathbf{p}}t}, && \leqslant b_{\mathbf{p}s}(t) \mid = \leqslant b_{\mathbf{p}s} \mid e^{-i\omega_{\mathbf{p}}t},\\
&\leqslant \quad c_{\mathbf{k}\lambda}(t) \mid = \leqslant c_{\mathbf{k}\lambda} \mid e^{-i\omega_{\mathbf{k}}t}, && \mid \overline{c}_{\mathbf{k}\lambda}(t) \geqslant = \mid \overline{c}_{\mathbf{k}\lambda} \geqslant e^{-i\omega_{\mathbf{k}}t}\\
&\mid \quad \underline{b}_{\mathbf{p}s}(t) \geqslant = \mid \underline{b}_{\mathbf{p}s} \geqslant e^{-i\omega_{\mathbf{p}}t}, && \leqslant \underline{a}_{\mathbf{p}s}(t) \mid = \leqslant \underline{a}_{\mathbf{p}s} \mid e^{i\omega_{\mathbf{p}}t}\\
&\leqslant \quad \underline{b}_{\mathbf{p}s}(t) \mid = \leqslant \underline{b}_{\mathbf{p}s} \mid e^{i\omega_{\mathbf{p}}t}, && \mid \underline{a}_{\mathbf{p}s}(t) \geqslant = \mid \underline{a}_{\mathbf{p}s} \geqslant e^{-i\omega_{\mathbf{p}}t},\\
&\mid \quad \overline{\underline{c}}_{\mathbf{k}\lambda}(t) \geqslant = \mid \overline{\underline{c}}_{\mathbf{k}\lambda} \geqslant e^{-i\omega_{\mathbf{k}}t}, && \leqslant \underline{c}_{\mathbf{k}\lambda}(t) \mid = \leqslant \underline{c}_{\mathbf{k}\lambda} \mid e^{i\omega_{\mathbf{k}}t} 3.10
\end{aligned} \tag{3.1}
$$

We call such operators as $I_{\mathbf{p}} \leqslant \underline{a}_{\mathbf{p}s}(t) \mid$ and $\mid \underline{c}_{\mathbf{k}\lambda}(t) \geqslant j_{\mathbf{k}}$ transformation operators. Such an operator as $\leqslant \underline{a}_{\mathbf{p}s}(t) \mid$ changes as time, and $\underline{I}_{\mathbf{p}}$ and $j_{\mathbf{k}}$ etc. do not change. In the Heisenberg picture the evolution of a quantum field system as time is carried by the field operators according to the equations of motion

$$
\dot{F} = -i\,[F,H] = -i\,[F,H_F]\,, \tag{3.11}
$$

$$
\dot{W} = i\,[W,H] = i\,[W,H_W]\,, \tag{3.12}
$$

where $F = \psi_0(x),\ \psi_0^+(x),\ A_{0\mu}(x),\ \pi_{0\mu}(x),\ \mid a_{\mathbf{p}s}(t) \geqslant,\ \mid b_{\mathbf{p}s}(t) \geqslant,\ \mid c_{\mathbf{k}\lambda}(t) \geqslant,\ \leqslant a_{\mathbf{p}s}(t) \mid,\ \leqslant b_{\mathbf{p}s}(t) \mid$ and $\leqslant c_{\mathbf{k}\lambda}(t) \mid$, $W = \underline{\psi}_0(x),\ \underline{\psi}_0^+(x),\ \underline{A}_{0\mu}(x),\ \underline{\pi}_{0\mu}(x),\ \mid \underline{a}_{\mathbf{p}s}(t) \geqslant,\ \mid \underline{b}_{\mathbf{p}s}(t) \geqslant,\ \mid \underline{c}_{\mathbf{k}\lambda}(t) \geqslant, \leqslant \underline{a}_{\mathbf{p}s}(t) \mid,\ \leqslant \underline{b}_{\mathbf{p}s}(t) \mid$ and $\leqslant \underline{c}_{\mathbf{k}\lambda}(t) \mid$. The equation (3.11) is well-known as the Heisenberg equation, while (3.12) is a new equation of motion. We will see

$$
[H_W, H_F] = [H_F, H] = [H_W, H] = 0, \tag{3.13}
$$

hence H_F and H_W are the constants of motion. Thus, from (3.11) – (3.13) we have

$$
F\,(t) = e^{iH_0 t} F\,(0)\, e^{-iH_0 t} = e^{iH_{F0} t} F\,(0)\, e^{-iH_{F0} t}, \tag{3.14}
$$

$$
W\,(t) = e^{-iH_0 t} W\,(0)\, e^{iH_0 t} = e^{-iH_{W0} t} W\,(0)\, e^{iH_{W0} t}, \tag{3.15}
$$

It will be proved that (3.11) − (3.12) are consistent with (2.11) – (2.12), and (3.14) − (3.15) are consistent with (3.10) .

3.1. ***Properties and multiplication rules of the transformation operators.*** 1. A transformation operator as $I_{\mathbf{p}} \leqslant \underline{a}_{\mathbf{p}s} \mid$ is regarded a whole, hence the order of its two parts cannot be exchanged, say, $I_{\mathbf{p}} \leqslant \underline{a}_{\mathbf{p}s} \mid$ and $\mid \overline{c}_{\mathbf{k}\lambda} \geqslant \underline{j}_{(-\mathbf{k})}$ cannot be written as $\leqslant \underline{a}_{\mathbf{p}s} \mid I_{\mathbf{p}}$ and $\underline{j}_{(-\mathbf{k})} \mid \overline{c}_{\mathbf{k}\lambda} \geqslant$, respectively.

2. Multiplication rules of such operators as $\leqslant a_{\mathbf{p}s} \mid$ and $\mid a_{\mathbf{p}s} \geqslant$.

Such an operator in the form $\leqslant a_{\mathbf{p}s} \mid$ is equivalent to an annihilation operator $a_{\mathbf{p}s}$, and such an operator in the form $\mid a_{\mathbf{p}s} \geqslant$ is equivalent to an creation operator $a^{+}_{\mathbf{p}s}$ in the conventional QED. Thus, let $\alpha = a$, b, $\underline{a}$ and $\underline{b}$, $\gamma = c$ and $\underline{c}$, the multiplication rules of the operators are

$$\begin{aligned} \leqslant & \ \alpha_{\mathbf{p}s}(t) \,\|\, \alpha_{\mathbf{p}'s'}(t) \geqslant \\ = & \ \leqslant \alpha_{\mathbf{p}s}(t) \mid \alpha_{\mathbf{p}'s'}(t) \geqslant - \mid \alpha_{\mathbf{p}'s'}(t) \geqslant \leqslant \alpha_{\mathbf{p}s}(t) \mid \\ = & \ \delta_{\alpha\alpha'}\delta_{\mathbf{p}\mathbf{p}'}\delta_{ss'} - \mid \alpha_{\mathbf{p}'s'}(t) \geqslant \leqslant \alpha_{\mathbf{p}s}(t) \mid, 3.16 \end{aligned} \tag{3.2}$$

$$\begin{aligned} \leqslant & \ \gamma_{\mathbf{k}\lambda}(t) \,\|\, \gamma'_{\mathbf{k}'\lambda'}(t) \geqslant \\ = & \ \leqslant \gamma_{\mathbf{k}\lambda}(t) \mid \gamma'_{\mathbf{k}'\lambda'}(t) \geqslant + \mid \gamma'_{\mathbf{k}'\lambda'}(t) \geqslant \leqslant \gamma_{\mathbf{k}\lambda}(t) \mid \\ = & \ \mid \gamma'_{\mathbf{k}'\lambda'}(t) \geqslant \leqslant \gamma_{\mathbf{k}\lambda}(t) \mid + \begin{cases} \delta_{\gamma\gamma'}\delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}, \lambda = 1,2,3, \\ -\delta_{\gamma\gamma'}\delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}, \lambda = 4 \end{cases} \ 3.17 \end{aligned} \tag{3.3}$$

$$\begin{aligned} \leqslant & \ \gamma_{\mathbf{k}\lambda}(t) \,\|\, \alpha_{\mathbf{p}'s'}(t) \geqslant = \mid \alpha_{\mathbf{p}'s'}(t) \geqslant \leqslant \gamma_{\mathbf{k}\lambda}(t) \mid, \\ \leqslant & \ \alpha_{\mathbf{p}s}(t) \,\|\, \gamma_{\mathbf{k}\lambda}(t) \geqslant = \mid \gamma_{\mathbf{k}\lambda}(t) \geqslant \leqslant \alpha_{\mathbf{p}s}(t) \mid 3.18 \end{aligned} \tag{3.4}$$

From (3.16)-(3.17) we have

$$\{\leqslant \alpha_{\mathbf{p}s}(t) \mid, \mid \alpha_{\mathbf{p}'s'}(t) \geqslant\} = \delta_{\alpha\alpha'}\delta_{\mathbf{p}\mathbf{p}'}\delta_{ss'} \tag{3.19}$$

$$, (3.20)$$

$$[\leqslant \gamma_{\mathbf{k}\lambda}(t) \mid, \mid \gamma'_{\mathbf{k}'\lambda'}(t) \geqslant] = \begin{cases} \delta_{\gamma\gamma'}\delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}, \lambda = 1,2,3, \\ -\delta_{\gamma\gamma'}\delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}, \lambda = 4 \end{cases} ,$$

The other commutators or anticommutators are all zero.

3. States and inner products of states.

(3.19) and (3.20) are the same as the anticommutation relations and the commutation relations of the conventional QED, respectiveky. As the conventional QED, from (3.19)-(3.20) we see free fermion states and n-photon states to be

$$\mid \alpha_{\mathbf{p}s}\rangle = \mid \alpha_{\mathbf{p}s} \geqslant \mid 0\rangle, \qquad \langle \alpha_{\mathbf{p}s} \mid = \langle 0 \mid \leqslant \alpha_{\mathbf{p}s} \mid . \tag{3.21}$$

$$\begin{aligned} \mid & \ n_{\mathbf{k}\lambda}\rangle \equiv 1\sqrt{n!}(\mid c_{\mathbf{k}\lambda} \geqslant)^n \mid 0\rangle, \ \langle n_{\mathbf{k}\lambda} \mid \equiv \langle 0 \mid (\leqslant c_{\mathbf{k}\lambda} \mid)^n 1\sqrt{n!}, \\ \mid & \ \underline{n}_{\mathbf{k}\lambda}\rangle \equiv 1\sqrt{n!}(\mid \underline{c}_{\mathbf{k}\lambda} \geqslant)^n \mid 0\rangle, \langle \underline{n}_{\mathbf{k}\lambda} \mid \equiv \langle 0 \mid (\leqslant \underline{c}_{\mathbf{k}\lambda} \mid)^n 1\sqrt{n!}.3.22 \end{aligned} \tag{3.5}$$

Considering (3.19)-(3.20) and

$$\begin{aligned} \leqslant & \ \alpha_{\mathbf{p}s} \mid 0\rangle = \leqslant \gamma_{\mathbf{k}\lambda} \mid 0\rangle = \langle 0 \mid \alpha_{\mathbf{p}s} \geqslant = \langle 0 \mid \gamma_{\mathbf{k}\lambda} \geqslant = 0, \\ \langle 0 & \ \mid \ 0\rangle = 1, 3.23 \end{aligned} \tag{3.6}$$

we obtain the inner products of states to be

$$\langle \alpha_{\mathbf{p}s} \mid \cdot \mid \alpha'_{\mathbf{p}'s'}\rangle = \langle \alpha_{\mathbf{p}s} \mid \alpha'_{\mathbf{p}'s'}\rangle = \delta_{\alpha\alpha'}\delta_{\mathbf{p}\mathbf{p}'}\delta_{ss'}, \tag{3.24}$$

(3.25)

$$\langle \gamma_{\mathbf{k}\lambda} \mid \cdot \mid \gamma'_{\mathbf{k}'\ \lambda'} \rangle = \langle \gamma_{\mathbf{k}\lambda} \mid \gamma'_{\mathbf{k}'\ \lambda'} \rangle = \begin{cases} \delta_{\gamma\gamma'}\delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}, \lambda = 1,2,3, \\ -\delta_{\gamma\gamma'}\delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}, \lambda = 4 \end{cases}$$

$$\langle \beta_{\mathbf{p}s} \mid \cdot \mid \gamma_{\mathbf{k}\lambda} \rangle = 0, \tag{3.26}$$

4. Inner products of $I_{\mathbf{p}}$, $J_{\mathbf{p}}$,$\underline{I}_{\mathbf{p}}$, and $\underline{J}_{\mathbf{p}}$.

The original form of the transformation operators is the same as $\mid \underline{a}_{\mathbf{p}s}\rangle \leqslant a_{\mathbf{p}s} \mid$, $\mid b_{{}_{\mathbf{p}s}} \geqslant \langle \underline{b}_{\mathbf{p}s} \mid$ or $\mid c_{\mathbf{k}\lambda}\rangle \leqslant \underline{c}_{\mathbf{k}\lambda} \mid$, and their original meanings are

$$\begin{array}{rl} & \mid \underline{a}_{\mathbf{p}s}\rangle \leqslant a_{\mathbf{p}s} \mid\mid a_{\mathbf{p}s}\rangle = \mid \underline{a}_{\mathbf{p}s}\rangle, \\ \langle b_{\mathbf{p}s} & \mid \mid b_{{}_{\mathbf{p}s}} \geqslant \langle \underline{b}_{\mathbf{p}s} \mid = \langle \underline{b}_{\mathbf{p}s} \mid \\ & \mid c_{\mathbf{k}\lambda}\rangle \leqslant \underline{c}_{\mathbf{k}\lambda} \mid\mid \underline{c}_{\mathbf{k}\lambda}\rangle = \mid c_{\mathbf{k}\lambda}\rangle .3.27 \end{array} \tag{3.7}$$

The transformation operators can also be written as

$$\begin{array}{l} \mid \underline{a}_{\mathbf{p}s}\rangle \leqslant a_{\mathbf{p}s}(t) \mid = \mid \underline{a}_{\mathbf{p}s}\rangle \underline{I}^{+}_{\mathbf{p}} \cdot \underline{I}_{\mathbf{p}} \leqslant a_{\mathbf{p}s}(t) \mid, \\ \mid b_{{}_{\mathbf{p}s}}(t) \geqslant \langle \underline{b}_{\mathbf{p}s} \mid = \mid b_{{}_{\mathbf{p}s}}(t) \geqslant \underline{I}_{(-\mathbf{p})} \cdot \underline{I}^{+}_{(-\mathbf{p})} \langle \underline{b}_{\mathbf{p}s} \mid, \\ \mid \underline{c}_{\mathbf{k}\lambda}\rangle \leqslant c_{\mathbf{k}\lambda}(t) \mid = \mid \underline{c}_{\mathbf{k}\lambda}\rangle \underline{J}^{+}_{\mathbf{k}} \cdot \underline{J}_{\mathbf{k}} \leqslant c_{\mathbf{k}\lambda}(t) \mid, \\ \mid b_{\mathbf{p}s}\rangle \leqslant \underline{b}_{\mathbf{p}s}(t) \mid = \mid b_{\mathbf{p}s}\rangle I^{+}_{\mathbf{p}} \cdot I_{\mathbf{p}} \leqslant \underline{b}_{\mathbf{p}s}(t) \mid, \\ \mid a_{\mathbf{p}s}\rangle \leqslant \underline{a}_{\mathbf{p}s}(t) \mid = \mid a_{\mathbf{p}s}\rangle I^{+}_{(-\mathbf{p})} \cdot I_{(-\mathbf{p})} \leqslant \underline{a}_{\mathbf{p}s}(t) \mid, \\ \mid c_{\mathbf{k}\lambda}\rangle \leqslant \underline{c}_{\mathbf{k}\lambda}(t) \mid = \mid c_{\mathbf{k}\lambda}\rangle J^{+}_{\mathbf{k}} \cdot J_{\mathbf{k}} \leqslant \underline{c}_{\mathbf{k}\lambda}(t) \mid .3.28 \end{array} \tag{3.8}$$

In orter to easily deal with problems, now we divide such an operator as $\mid \underline{a}_{\mathbf{p}s}\rangle \leqslant a_{\mathbf{p}s}(t) \mid$ into two parts $\mid \underline{a}_{\mathbf{p}s}\rangle \underline{I}^{+}_{\mathbf{p}}$ and $\underline{I}_{\mathbf{p}} \leqslant a_{\mathbf{p}s}(t) \mid$, leave such a part as $\mid \underline{\alpha}_{\mathbf{p}s}\rangle \underline{I}^{+}_{\mathbf{p}}$ belonging a coupling operator or a mass operator (see below), and leave such a part as $\underline{I}_{\mathbf{p}} \leqslant \alpha_{\mathbf{p}s} \mid$ belonging a field operator and also call it a transformation operator.

Let $K = I_{\mathbf{p}}$, $I^{+}_{\mathbf{p}}$, $\underline{I}_{\mathbf{p}}$, $\underline{I}^{+}_{\mathbf{p}}$, $J_{\mathbf{k}}$ and $\underline{J}_{\mathbf{k}}$. Their properties and the multiplication rules are as follows.

$$J_{-\mathbf{k}} = J^{+}_{\mathbf{k}}, \qquad \underline{J}_{-\mathbf{k}} = \underline{J}^{+}_{\mathbf{k}} \tag{3.29}$$

$$\begin{array}{rcl} I^{+}_{\mathbf{p}'} \cdot I_{\mathbf{p}} & = & I_{\mathbf{p}} \cdot I^{+}_{\mathbf{p}'} = \underline{I}^{+}_{\mathbf{p}'} \cdot \underline{I}_{\mathbf{p}} = \underline{I}_{\mathbf{p}} \cdot \underline{I}^{+}_{\mathbf{p}'} = \delta_{\mathbf{p}\mathbf{p}'} \\ J^{+}_{\mathbf{k}'} \cdot J_{\mathbf{k}} & = & \underline{J}^{+}_{\mathbf{k}'} \cdot \underline{J}_{\mathbf{k}} = \delta_{\mathbf{k}\mathbf{k}'}, \\ & & \textit{The other inner products are all zero.} 3.30 \end{array} \tag{3.9}$$

$I_{\mathbf{p}}$, $I^{+}_{\mathbf{p}}$, $\underline{I}^{+}_{\mathbf{p}}$, $\underline{I}_{\mathbf{p}}$, $J_{\mathbf{k}}$, $\underline{J}_{\mathbf{k}}$ can be regarded a base vector of $I_{\mathbf{p}} - space$, $\cdots$ a base vector of $\underline{J}_{\mathbf{k}} - space$, respectively. $I_{\mathbf{p}} - space$ and $I^{+}_{\mathbf{p}} - space$ are dual spaces; $\underline{I}_{\mathbf{p}} - space$ and $\underline{I}^{+}_{\mathbf{p}} - space$ are dual spaces; $J_{\mathbf{k}} - space$ and $\underline{J}_{\mathbf{k}} - space$ are self-dual space.

5.When multipling a transformation operator by other transformation operator, we define the product to be such an operator obtained after achieving multiplication of the two $K's$, i.e.,

$$
\begin{aligned}
AK_A \cdot K_B B &\equiv (AK_A)\cdot(K_B B) \equiv A(K_A \cdot K_B)B, \\
K_B B \cdot\ AK_A &\equiv (K_B B)\cdot\ (AK_A) \equiv \pm A(K_A \cdot K_B)B, \\
K_B B \cdot K_A A &= (K_B B)\cdot(K_A A) = (K_B \cdot K_A)BA, \\
BK_B \cdot AK_A &= (BK_B)\cdot(AK_A) \quad = BA(K_B \cdot K_A), \\
A \cdot K &= 0, \quad AK = KA, 3.31
\end{aligned}
\tag{3.10}
$$

where $A,\ B = \leqslant a_{\mathbf{p}s}(t) \mid$ etc. or $\mid b_{\mathbf{p}s}\rangle$ etc., when bath A and B are fermion operators, the second formula in (3.31) takes '-', and otherwise takes '+', e.g.,

$$
\begin{aligned}
\underline{I}_{\mathbf{p}'} &\leqslant b_{\mathbf{p}'s'} \mid \cdot \mid b_{\mathbf{p}s} \geqslant \underline{I}_{\mathbf{p}} = - \mid b_{\mathbf{p}s} \geqslant \underline{I}_{\mathbf{p}} \cdot \underline{I}_{\mathbf{p}'} \leqslant b_{\mathbf{p}'s'} \mid \\
&= - \mid b_{\mathbf{p}s} \geqslant\leqslant b_{\mathbf{p}'s'} \mid \delta_{\mathbf{p}\mathbf{p}'}\delta_{ss'},
\end{aligned}
$$

$$
\begin{aligned}
\underline{j}_{\mathbf{k}} &\leqslant c_{\mathbf{k}\lambda} \mid \cdot \mid \overline{c}_{\mathbf{k}'\lambda'} \geqslant \underline{j}_{(-\mathbf{k}')} = \mid \overline{c}_{\mathbf{k}'\lambda'} \geqslant \underline{j}_{(-\mathbf{k}')} \cdot \underline{j}_{\mathbf{k}} \leqslant c_{\mathbf{k}\lambda} \mid \\
&= \mid \overline{c}_{\mathbf{k}'\lambda'} \geqslant\leqslant c_{\mathbf{k}\lambda} \mid \delta_{\mathbf{k}\mathbf{k}'},
\end{aligned}
$$

$$
\underline{I}_{\mathbf{p}} \leqslant a_{\mathbf{p}s} \mid \cdot \underline{I}_{\mathbf{p}'} \leqslant a_{\mathbf{p}'s'} \mid = \underline{I}_{\mathbf{p}} \cdot \underline{I}_{\mathbf{p}'} \leqslant a_{\mathbf{p}s} \mid \leqslant a_{\mathbf{p}'s'} \mid = 0,
$$

$$
\begin{aligned}
\underline{j}_{\mathbf{k}} &\leqslant c_{\mathbf{k}\lambda} \mid \cdot \underline{j}_{\mathbf{k}'} \leqslant c_{\mathbf{k}'\lambda'} \mid = \underline{j}_{\mathbf{k}} \cdot \underline{j}_{\mathbf{k}'} \leqslant c_{\mathbf{k}'\lambda'} \mid \leqslant c_{\mathbf{k}\lambda} \mid \\
&= \delta_{\mathbf{k}(-\mathbf{k}')} \leqslant c_{\mathbf{k}'\lambda'} \mid \leqslant c_{\mathbf{k}\lambda} \mid,
\end{aligned}
$$

where the sign '$\cdot$' represents to make the inner product. KA represents the operator A does not act to K. K can be constructed by states (see appendix A). We call an operator with $\mid \alpha \geqslant$ or $\leqslant \alpha \mid$ a F-operator, and an operator with $\mid \underline{\alpha} \geqslant$ or $\leqslant \underline{\alpha} \mid$ a W-operator. According to the definition above, a product of two F-operators (or two W-operators) is still a F-operators (or a W-operators). It can be seen from (3.30) that the product of a F-transformation operator and a W-transformation operator must be equal to zero.

When many operators containing K multiply, associative law does no longer hold water. Hence we must appoint the associative order of the operators. This order is easily appointed because the matter, in fact, appears in only deriving Lagrangian density and Hamiltan density.

It is obvious that an essential difference between transformation operators and the creation or annihilation operators is the factor K .

6. From (3.19)-(3.20),(3.30)-(3.31) and (3.1)-(3.8), we easily derive the commutation or anticommutation relations of the transformation operators and the field operators.

$$
\begin{aligned}
\{\underline{I}_{\mathbf{p}} &\leqslant a_{\mathbf{p}s}(t) \mid, \mid a_{\mathbf{p}'s'}(t) \geqslant \underline{I}^{+}_{\mathbf{p}'}\} \\
&= \{\underline{I}^{+}_{-\mathbf{p}} \leqslant b_{\mathbf{p}s}(t) \mid, \mid b_{\mathbf{p}'s'}(t) \geqslant \underline{I}_{-\mathbf{p}'}\} \\
&= [\underline{J}_{\mathbf{k}} \leqslant c_{\mathbf{k}\lambda}(t) \mid, \mid \overline{c}_{\mathbf{k}'\lambda'}(t) \geqslant \underline{J}^{+}_{\mathbf{k}'}] = 0, 3.32
\end{aligned}
\tag{3.11}
$$

$$
\begin{aligned}
\{I^{+}_{-\mathbf{p}} &\leqslant \underline{a}_{\mathbf{p}s}(t)\mid,\mid \underline{a}_{\mathbf{p}'s'}(t)\geqslant I_{-\mathbf{p}'}\} \\
&= \{I_{\mathbf{p}}\leqslant \underline{b}_{\mathbf{p}s}(t)\mid,\mid \underline{b}_{\mathbf{p}'s'}(t)\geqslant I^{+}_{\mathbf{p}'} \\
&= [J_{\mathbf{k}}\leqslant \underline{c}_{\mathbf{k}\lambda}(t)\mid,\mid \overline{c}_{\mathbf{k}'\lambda'}(t)\geqslant J^{+}_{\mathbf{k}'}] = 0, 3.33
\end{aligned} \tag{3.12}
$$

$$
\{\psi_\alpha(\mathbf{x},t),\psi^{+}_\beta(\mathbf{y},t)\} = \{\psi_\alpha(\mathbf{x},t),\psi_\beta(\mathbf{y},t)\} = \{\psi^{+}_\alpha(\mathbf{x},t),\psi^{+}_\beta(\mathbf{y},t)\} = 0, \tag{3.34}
$$

$$
[A_\mu(\mathbf{x},\mathbf{t}),\pi_\nu(\mathbf{y},\mathbf{t})]=[A_\mu(\mathbf{x},\mathbf{t}),A_\nu(\mathbf{y},\mathbf{t})]=[\pi_\mu(\mathbf{x},\mathbf{t}),\pi_\nu(\mathbf{y},\mathbf{t})] = 0, \tag{3.35}
$$

$$
\{\underline{\psi}_\alpha(\mathbf{x},t),\underline{\psi}^{+}_\beta(\mathbf{y},t)\} = \{\underline{\psi}_\alpha(\mathbf{x},t),\underline{\psi}_\beta(\mathbf{y},t)\} = \{\underline{\psi}^{+}_\alpha(\mathbf{x},t),\underline{\psi}^{+}_\beta(\mathbf{y},t)\} = 0, \tag{3.36}
$$

$$
[\underline{A}_\mu(\mathbf{x},\mathbf{t}),\underline{\pi}_\nu(\mathbf{y},\mathbf{t})]=[\underline{A}_\mu(\mathbf{x},\mathbf{t}),\underline{A}_\nu(\mathbf{y},\mathbf{t})]=[\underline{\pi}_\mu(\mathbf{x},\mathbf{t}),\underline{\pi}_\nu(\mathbf{y},\mathbf{t})] = 0. \tag{3.37}
$$

The others are all zero as well. The commutation or anticommutation relations are diferent from those of the convertional QED.

4. *The energies and charges of particles.*

From (2.7) – (2.9) , (3.1) – (3.8) and (3.29) – (3.31) we obtain

$$
\begin{aligned}
H_{F0} &= \sum_{\mathbf{p}s} E_{\mathbf{p}}\left(\mid a_{\mathbf{p}s}\geqslant\leqslant a_{\mathbf{p}s}\mid + \mid b_{\mathbf{p}s}\geqslant\leqslant b_{\mathbf{p}s}\mid\right) \\
&+\sum_{\mathbf{k}}\omega_{\mathbf{k}}\Big(\sum_{\lambda=1}^{3}\mid c_{\mathbf{k}\lambda}\geqslant\leqslant c_{\mathbf{k}\lambda}\mid - \mid c_{\mathbf{k}4}\geqslant\leqslant c_{\mathbf{k}4}\mid\Big), 4.1
\end{aligned} \tag{4.1}
$$

$$
\begin{aligned}
H_{W0} &= \sum_{\mathbf{p}s} E_{\mathbf{p}}\left(\mid \underline{b}_{\mathbf{p}s}\geqslant\leqslant \underline{b}_{\mathbf{p}s}\mid + \mid \underline{a}_{\mathbf{p}s}\geqslant\leqslant \underline{a}_{\mathbf{p}s}\mid\right) \\
&+\sum_{\mathbf{k}}\omega_{\mathbf{k}}\Big(\sum_{\lambda=1}^{3}\mid \underline{c}_{\mathbf{k}\lambda}\geqslant\leqslant \underline{c}_{\mathbf{k}\lambda}\mid - \mid \underline{c}_{\mathbf{k}4}\geqslant\leqslant \underline{c}_{\mathbf{k}4}\mid\Big), 4.2
\end{aligned} \tag{4.2}
$$

$$
Q_F = \sum_{\mathbf{p}s}\left(\mid a_{\mathbf{p}s}\geqslant\leqslant a_{\mathbf{p}s}\mid - \mid b_{\mathbf{p}s}\geqslant\leqslant b_{\mathbf{p}s}\mid\right), \tag{4.3}
$$

$$
Q_W = \sum_{\mathbf{p}s}\left(-\mid \underline{b}_{\mathbf{p}s}\geqslant\leqslant \underline{b}_{\mathbf{p}s}\mid + \mid \underline{a}_{\mathbf{p}s}\geqslant\leqslant \underline{a}_{\mathbf{p}s}\mid\right). \tag{4.4}
$$

From (4.1) and (4.2) we see that energies are positive-definite and (3.11) and (3.12) are consistent with (3.10), respectively.It is easily seen from (4.1) – (4.4) that

$$
\langle \underline{\sigma}\mid H_0\mid \underline{\sigma}\rangle = \langle \sigma\mid H_0\mid \sigma\rangle, \tag{4.5}
$$

$$\langle \underline{\sigma} \mid Q \mid \underline{\sigma} \rangle = \langle \sigma \mid Q \mid \sigma \rangle, \tag{4.6}$$

where $\sigma = a_{\mathbf{p}s},\ b_{\mathbf{p}s},\ c_{\mathbf{k}\lambda}$.

The Hamilton and charge operators can also be written as

$$H_{F0} = \int d^3x : [\psi'^{+}\gamma_4 \left(\gamma_j \partial_j + m\right) \psi' + 12 \left(\dot{A'}_\mu \dot{A'}_\mu + \partial_j A'_\nu \partial_j A'_\nu \right)] \ :, \tag{4.7}$$

$$H_{W0} = -\int d^3x : [(\underline{\psi}'^{+}\gamma_4(\gamma_j \partial_j + m)\underline{\psi}' - 12(\dot{\underline{A}}'_\mu \dot{\underline{A}}'_\mu + \partial_j \underline{A}'_\nu \partial_j \underline{A}'_\nu)] :, \tag{4.8}$$

$$Q_F = \int d^3x : \psi'^{+}\psi' :, \qquad Q_F = \int d^3x : \underline{\psi}'^{+}\underline{\psi}' :, \tag{4.9}$$

where the double-dot notation $: \cdots :$ is known as normal ordering. An operator product is in normal ordered form if all operators as $\mid \alpha \geqslant$ stand to the left of all operators as $\leqslant \alpha \mid$. In contrast with the conventional QED, (4.1)-(4.4) or (4.7)-(4.9) are the inferences of the multiplication rules, and are not definition. In (4.7)-(4.9),

$$\psi'_0\left(x\right) = 1\sqrt{V} \sum_{\mathbf{p}s} \left(\leqslant a_{\mathbf{p}s}(t) \mid u_{\mathbf{p}s} e^{i\mathbf{px}} + \mid b_{\mathbf{p}s}(t) \geqslant v_{\mathbf{p}s} e^{-i\mathbf{px}} \right), \tag{4.10}$$

$$A'_{0\mu}\left(x\right) = 1\sqrt{V} \sum_{\mathbf{k}} 1\sqrt{2\omega_{\mathbf{k}}} \sum_{\lambda=1}^{4} e^{\lambda}_{\mathbf{k}\mu} \left(\leqslant c_{\mathbf{k}\lambda}(t) \mid e^{i\mathbf{kx}} + \mid \overline{c}_{\mathbf{k}\lambda}(t) \geqslant e^{-i\mathbf{kx}} \right), \tag{4.11}$$

$$\underline{\psi'}_0\left(x\right) = 1\sqrt{V} \sum_{\mathbf{p}s} \left(\mid \underline{b}_{\mathbf{p}s}(t) \geqslant u_{\mathbf{p}s} e^{i\mathbf{px}} + \leqslant \underline{a}_{\mathbf{p}s}(t) \mid v_{\mathbf{p}s} e^{-i\mathbf{px}} \right), \tag{4.12}$$

$$\underline{A}'_\mu\left(x\right) = 1\sqrt{V} \sum_{\mathbf{k}} 1\sqrt{2\omega_{\mathbf{k}}} \sum_{\lambda=1}^{4} e^{\lambda}_{\mathbf{k}\mu} \left(\mid \overline{\underline{c}}_{\mathbf{k}\lambda}(t) \geqslant e^{i\mathbf{kx}} + \leqslant \underline{c}_{\mathbf{k}\lambda}(t) \mid e^{-i\mathbf{kx}} \right), \tag{4.13}$$

$$\pi'_{0\psi} = i\psi'^{+}_{0(x)} = i\sqrt{V} \sum_{\mathbf{p}s} \left(\mid a_{\mathbf{p}s}(t) \geqslant u^{+}_{\mathbf{p}s} e^{-i\mathbf{px}} + \leqslant b_{\mathbf{p}s}(t) \mid v^{+}_{\mathbf{p}s} e^{i\mathbf{px}} \right) \tag{4.14}$$

(4.15)

$$\pi'_{0\mu} = \dot{A}'_{0\mu}(x) = -i\sqrt{V}\sum_{\mathbf{k}}\sqrt{\omega_{\mathbf{k}}2}\sum_{\lambda=1}^{4} e^{\lambda}_{\mathbf{k}\mu}\left(\leqslant c_{\mathbf{k}\lambda}(t) \mid e^{i\mathbf{kx}} - \mid \overline{c}_{\mathbf{k}\lambda}(t) \geqslant e^{-i\mathbf{kx}}\right),$$

$$\pi'_{0\psi} = -i\underline{\psi}_0^{\prime +}(x) = -i\sqrt{V}\sum_{\mathbf{p}s}\left(\leqslant \underline{b}_{\mathbf{p}s}(t) \mid u^{+}_{\mathbf{p}s}e^{-i\mathbf{px}} + \mid \underline{a}_{\mathbf{p}s}(t) \geqslant v^{+}_{\mathbf{p}s}e^{i\mathbf{px}}\right), \tag{4.16}$$

$$\underline{\pi}'_{0\mu} = \underline{\dot{A}}'_{0\mu} = -i\sqrt{V}\sum_{\mathbf{k}}\sqrt{\omega_{\mathbf{k}}2}\sum_{\lambda=1}^{4} e^{\lambda}_{\mathbf{k}\mu}\left(\mid \underline{\overline{c}}_{\mathbf{k}\lambda}(t) \geqslant e^{i\mathbf{kx}} - \leqslant \underline{c}_{\mathbf{k}\lambda}(t) \mid e^{-i\mathbf{kx}}\right), \tag{4.17}$$

where the operators $\mid a_{\mathbf{p}s}(t) \geqslant$ etc., are the same as (3.10). From (3.19)-(3.20) and (4.10)-(4.17) we have

$$\{\psi'_{\alpha}(\mathbf{x},t), \psi'^{+}_{\beta}(\mathbf{y},t)\} = \{\underline{\psi}'_{\alpha}(\mathbf{x},t), \underline{\psi}'^{+}_{\beta}(\mathbf{y},t)\} = \delta(\mathbf{x}-\mathbf{y})\delta_{\alpha\beta}, \tag{4.18}$$

$$[A'_{\mu}(\mathbf{x},t), \pi'_{\nu}(\mathbf{y},t)] = [\underline{A}'_{\mu}(\mathbf{x},t), \underline{\pi}'_{\nu}(\mathbf{y},t)] = i\delta(\mathbf{x}-\mathbf{y})\delta_{\mu\nu}. \tag{4.19}$$

All other anticommutators for fermion fields and commutattors for photon fields and fermion fields and photon fields are zero. (4.18)-(4.19) are the same as those in the convintional QED.

5. *Subsidiary condition*

After the Maxwell field is quantized, the Lorentz condition (2.13) is no longer applicable. From (4.11) and (4.13) we have

$$\left(\partial_{\mu}A'_{\mu}\right)^{+} = i\sqrt{V}\sum_{\mathbf{k}} \mid \mathbf{k} \mid \sqrt{2\omega_{\mathbf{k}}}(\leqslant c_{\mathbf{k}3} \mid -i \leqslant c_{\mathbf{k}4} \mid)e^{ikx}, \tag{5.1}$$

$$\left(\partial_{\mu}\underline{A}'_{\mu}\right)^{-} = -i\sqrt{V}\sum_{\mathbf{k}} \mid \mathbf{k} \mid \sqrt{2\omega_{\mathbf{k}}}(\leqslant \underline{c}_{\mathbf{k}3} \mid -i \leqslant \underline{c}_{\mathbf{k}4} \mid)e^{-ikx}, \tag{5.2}$$

Thus we define the subsidiary condition to be

$$\left(\partial_{\mu}A'_{\mu}\right)^{+} \mid c_p\rangle = 0, \tag{5.3}$$

(5.4) $$\left(\partial_\mu \underline{A}'_\mu\right)^- \mid \underline{c}_p\rangle = 0.$$

$\mid c_p\rangle$ and $\mid \underline{c}_p\rangle$ are known as F-physics state ket and W-physics state ket, respectively. From (5.1) – (5.4) we obtain

(5.5)
$$\mid c_p\rangle = \mid c_T\rangle\{1 + \sum_{\mathbf{k}} f(\mathbf{k}) \mid c_{p\mathbf{k}}\rangle + \cdots + \sum_{\mathbf{k}_1\cdots\mathbf{k}_n} f(\mathbf{k}_1 \cdots \mathbf{k}_n) \mid c_{p\mathbf{k}_1}\rangle \cdots \mid c_{p\mathbf{k}_n}\rangle\},$$

$$\mid \underline{c}_p\rangle = \mid \underline{c}_T\rangle\{1 + \sum_{\mathbf{k}} \underline{f}(\mathbf{k}) \mid \underline{c}_{p\mathbf{k}}\rangle + \cdots$$

(5.1) $$+ \sum_{\mathbf{k}_1\cdots\mathbf{k}_n} \underline{f}(\mathbf{k}_1 \cdots \mathbf{k}_n) \mid \underline{c}_{p\mathbf{k}_1}\rangle \cdots \mid \underline{c}_{p\mathbf{k}_n}\rangle\}, 5.6$$

where $\mid c_T\rangle$ and $\mid \underline{c}_T\rangle$ are states containing only transverse photrons, and

(5.7) $$\mid c_{p\mathbf{k}}\rangle = \mid c_{\mathbf{k}3}\rangle + i \mid c_{\mathbf{k}4}\rangle,$$

(5.8) $$\mid \underline{c}_{p\mathbf{k}}\rangle = \mid \underline{c}_{\mathbf{k}3}\rangle + i \mid \underline{c}_{\mathbf{k}4}\rangle.$$

From (4.1)-(4.2) and (5.5)-(5.8) we obtain

(5.2)
$$\begin{aligned} \langle c_{p\mathbf{k}} \mid c_{p\mathbf{k}'}\rangle &= \langle \underline{c}_{p\mathbf{k}} \mid \underline{c}_{p\mathbf{k}'}\rangle \\ &= \langle c_{p\mathbf{k}} \mid H_0 \mid c_{p\mathbf{k}}\rangle = \langle \underline{c}_{p\mathbf{k}} \mid H_0 \mid \underline{c}_{p\mathbf{k}}\rangle = 0, 5.9 \end{aligned}$$

(5.10) $$\langle c_p \mid c_{p'}\rangle = \langle c_T \mid c_{T'}\rangle,\ \langle \underline{c}_p \mid \underline{c}_{p'}\rangle = \langle \underline{c}_T \mid \underline{c}_{T'}\rangle,$$

(5.3)
$$\begin{aligned} \langle c_p \mid H_0 \mid c_p\rangle &= \langle c_T \mid H_0 \mid c_T\rangle, \\ \langle \underline{c}_p \mid H_0 \mid \underline{c}_p\rangle &= \langle \underline{c}_T \mid H_0 \mid \underline{c}_T\rangle, 5.11 \end{aligned}$$

6. *The equations of motion*

From (3.1) -(3.4) , (3.6) , (3.8) , (4.1) and (4.2) , we have

(6.1) $$i\partial\psi_0\partial t = [\psi_0, H_{F0}] = \hat{H}_0\psi_0,$$